**Appendix B: Derivation of the object-image mapping relationship for a double-telecentric optical system**

The relationship between the vectors and can be described using homogeneous coordinates as follows.

(A2-1)

Where *P* represents image points, *G* denotes object points, and *Imodel*(·) indicates a specific mathematical transformation. The implementation process of expression (A2-1) can be divided into the following steps.

**Step1:** Transformation from to . The rotation matrix and the translation vector in the transformation process from *O*-*XYZ* to *OC-XCYCZC* are denoted as and **t**3×1, respectively. The rotation angles *θX* ​, *θY* ​, and *θZ*​ define the orientation of the *Z*-axis of *O*-*XYZ* within *OC-XCYCZC*. The relationship between and satisfies the following equation.

(A2-2)

Where 3×3 denotes the rotation matrix that is a function of the rotation angles, i.e., . Since the *ZC*-axis aligns with the optical axis *₯*, there are multiple ways to implement the rotation for 3×3. In this article, the X-Y-Z rotation convention is adopted, and 3×3 can be equivalently expressed as where ​, ​, and ​ represent the rotation matrices around the *XC*, *YC*, and *ZC* axes, respectively. Their forms are as follows.

, ,

**Step2:**Transformation from to . If we disregard the distortions of the optical system, the imaging of a telecentric optical system only involves scaling along the *xs*​ and *ys*​ axes, representing an orthogonal projection. Thus, let the magnification factor be , and the relationship is given by

(A2-3)

**Step3:** Transformation from to . The relationship between the two vectors is described by the following equation.

(A2-4)

Where (*x*0, *y*0) represents the coordinates of the origin *os*​ in the coordinate system *oμ-xμyμ*, and *sx* and *sy*​ represent the sensor pixel sizes along the *xs*​ and *ys*​ directions respectively. Typically, *sx*​ and *sy* are equal, and *s*0​ is used to denote both values, i.e., *sx*=*sy*=*s*0​. Therefore, Eq. (A2-4) can be rewritten as

(A2-5)

Let ​​, where ​ represents the mapping ratio between the pixel and its length in the object space, commonly referred to as the pixel ratio.

**Step4:** Transformation from to . By combining Equations (A2-2) to (A2-5), we can derive the transformation relationship between the vectors and as follows.

(A2-6)